Program : **B.Tech**

Subject Name: **Structural Design and Drawing (RCC-I)**

Subject Code: **CE-601**

Semester: **6th**

Unit – II

Design of Beams: Doubly reinforced rectangular & Flanged Beams, Lintel, Cantilever, simply supported and continuous beams, Beams with compression reinforcement: Redistribution of moments in continuous beams, Circular girders: Deep beams. Design of beam for shear and bond.

Introduction:

A beam experiences flexural stresses and shear stresses. It deforms and cracks are developed. ARC beam should have perfect bond between concrete and steel for composites action. It is primarily designed as flexural member and then checked for other parameters like shear, bond, deflection etc. In reinforced concrete beams, in addition to the effects of shrinkage, creep and loading history, cracks developed in tension zone effects its behaviour. Elastic design method (WSM) do not give a clear indication of their potential strengths. Several investigators have published behaviour of RC members at ultimate load. Ultimate strength design for beams was introduced into both the American and British code in 1950's. The Indian code ES456 introduced the ultimate state method of design in 1964. Considering both probability concept and ultimate load called as "Limit state method of design" was introduced in Indian code from 1978.

Behaviour of Reinforced concrete beam

To understand the behaviour of beam under transverse loading, a simply supported beam subjected to two point loading as shown in Fig. 2.1 is considered. This beam is of rectangular cross-section and reinforced at bottom.

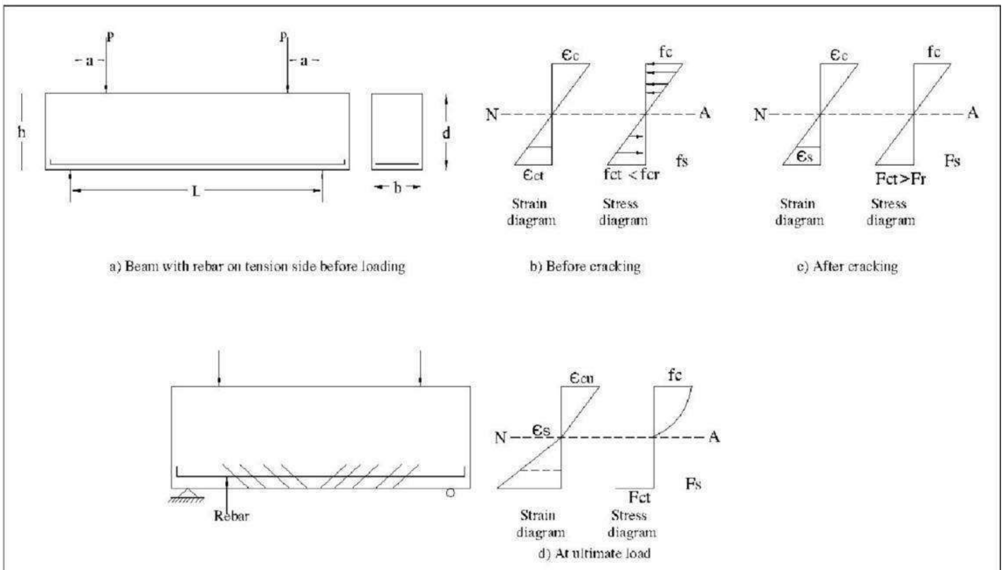

Fig 2.1 Behaviour of RC beam

When the load is gradually increased from zero to the ultimate load value, several stages of behaviour can be observed. At low loads where maximum tensile stress is less than modulus of rupture of concrete, the entire concrete is effective in resisting both compressive stress and tensile stress. At this stage, due to bonding tensile stress is also induced in steel bars.

With increase in load, the tensile strength of concrete exceeds the modulus of rupture of concrete and concrete cracks. Cracks propagate quickly upward with increase in loading up; to neutral axis. Strain and stress distribution across the depth is shown in Fig 4.1c. Width of crack is small. Tensile stresses developed are absorbed by steel bars. Stress and strain are proportional till fc<___. Further increase in load, increases strain and stress in the section and are no longer proportional. The distribution of stress – strain curve of concrete. Fig 41d shows the stress distribution at ultimate load.

Failure of beam depends on the amount of steel present in tension side. When moderate amount of steel is present, stress in steel reaches its yielding value and stretches a large amount with tension crack in concrete widens. Cracks in concrete propagate upward with increases in deflection of beam. This induces crushing of concrete in compression zone and called as "secondary compression failure". This

failure is gradual and is preceded by visible signs of distress. Such sections are called "under reinforced" sections.

When the amount of steel bar is large or very high strength steel is used, compressive stress in concrete reaches its ultimate value before steel yields. Concrete fails by crushing and failure is sudden. This failure is almost explosive and occur without warning. Such reactions are called "over reinforced section"

If the amount of steel bar is such that compressive stress in concrete and tensile stress in steel reaches their ultimate value simultaneously, then such reactions are called "Balanced Section".

The beams are generally reinforced in the tension zone. Such beams are termed as "singly reinforced" section. Some times rebars are also provided in compression zone in addition to tension rebars to enhance the resistance capacity, then such sections are called "Doubly reinforce section.

Assumptions

Following assumptions are made in analysis of members under flexure in limit state method

Plane sections normal to axis remain plane after bending. This implies that strain is proportional to the distance from neutral axis.

Maximum strain in concrete of compression zone at failure is 0.0035 in bending.

Tensile strength of concrete is ignored.

The stress-strain curve for the concrete in compression may be assumed to be rectangle, trapezium, parabola or any other shape which results in prediction of strength in substantial agreement with test results. Design curve given in IS456-2000 is shown in Fig. 2.2

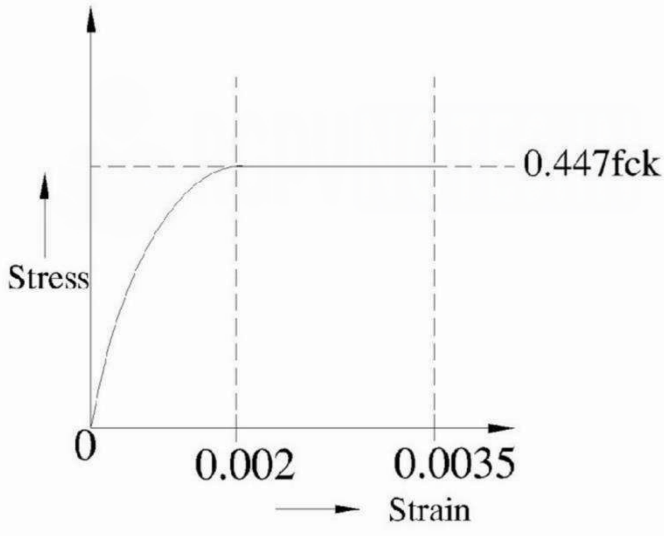

Fig 2.2 Stress-Strain Curve for Concrete

Stress – strain curve for steel bar with definite yield print and for cold worked deformed bars is shown respectively.

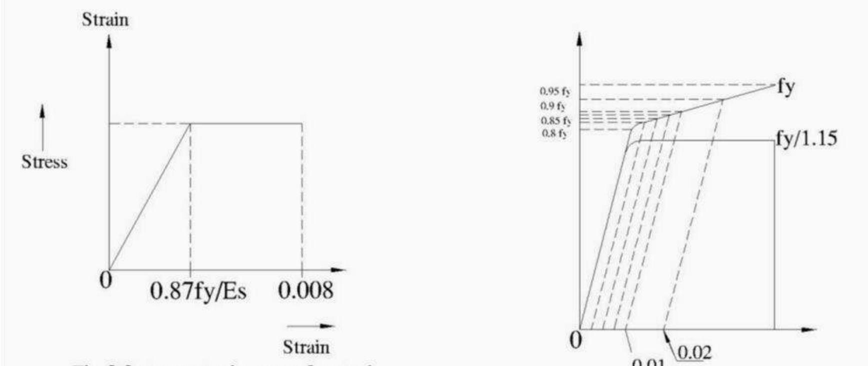

Fig 2.3 stress-strain curve for steel bar with defective yield point

Fig 2.4 stress-strain curve for cold worked deformed bars

To ensure ductility, maximum strain in tension reinforcement shall not be less than. + 0.002.

Perfect bond between concrete and steel exists.

Analysis of singly reinforced rectangular sections

Consider a rectangular section of dimension b x h reinforced with A_{st} amount of steel on tension side with effective cover Ce from tension extreme fiber to C.G of steel. Then effective depth d=h-ce, measured from extreme compression fiber to C.G of steel strain and stress distribution across the section is shown in Fig.2.4. The stress distribution is called stress block.

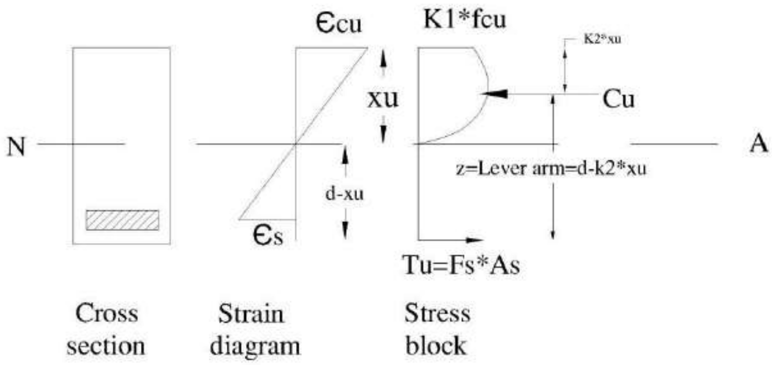

Fig 2.5 Stress Block

From similar triangle properly applied to strain diagram

$$\frac{\varepsilon cu}{xu} = \frac{\varepsilon s}{d - xu} \rightarrow (1)$$

$$\in s = \in cu \times \frac{d - xu}{xu} \rightarrow (2)$$

For the known value of x4 & 6cu the strain in steel is used to get the value of stress in steel from stress-strain diagram. Equation 4.4-1 can be used to get the value of neutral axis depth as

$$xu = \frac{\in cu}{\in s} \times (d - xu) = \frac{\in cu}{\in s} \times d - \frac{\in cu}{\in s} \times$$

$$1 + \left(\frac{\in cu}{\in s}\right) = \frac{\in cu}{\in s} \times$$

$$\therefore \quad xu\left(\frac{\in s+\in cu}{\in s}\right) = \frac{\in cu}{\in s} \times d$$

$$xu = \frac{\in cu}{\in cu+\in s} \times d - (3)$$

Here $\frac{\in cu}{\in cu+\in s}$ is called neutral axis factor

For equilibrium Cu = Tu.

$K_1, k_3 f_{cu}\, b\, x_u = f_s A_s$

$$\therefore fs = \frac{K1,k3fcubxu}{As} = \frac{k1k3fcub}{As} \times \frac{\in cu}{\in cu+\in s}$$

$$fs = k1k3fcu \times \frac{Ecu}{Ecu+Es} \times \frac{bd}{As} \quad Let\ p = steel\ raio = \frac{As}{bd}$$

$$\therefore fs = \frac{k1k3fcu}{p} \times \frac{Ecu}{Ecn+Es}\ or\ \frac{Ecu}{Ecu+Es} = \frac{fsp}{k1k3fcu} - (4)$$

Value of fs can be graphically computed for a given value of P as shown in Fig2.6

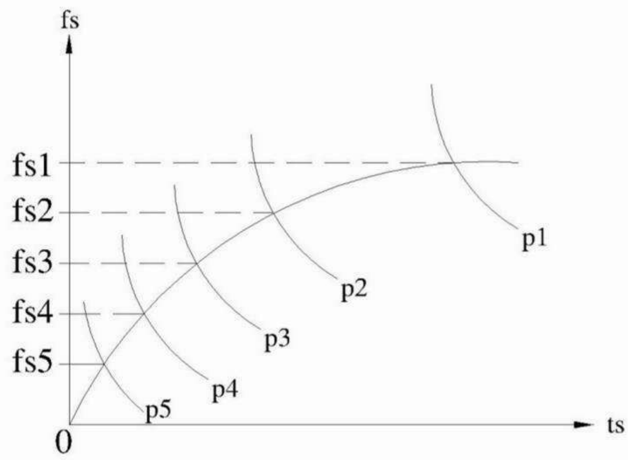

Fig 2.6

After getting fs graphically, the ultimate moment or ultimate moment of resistance is calculated as

$Mu = T_u \times Z = f_s A_s(d-k_2xu)$

$Mu = Cu \times Z = k_1k_2f_{cu}bx_u \times (d-k_2xu)$

Consider

$$Mu = f_s A_s\left(d - k2 \times \frac{\epsilon cu}{\epsilon cu+\epsilon s} d\right) = fsAsd\left(1 - kz \frac{\epsilon cu}{\epsilon cu+\epsilon s}\right)$$

From (4) $$\frac{Ecu}{Ecu+Es} = \frac{f_s\,p}{k1k3fcm}$$

$$\therefore Mu = fsAsd\left(1 - \frac{k2fsp}{k1k3fcu}\right) - (5)$$

Here the term $1 - \frac{k2fsp}{k1k3fcu}$ is called lever arm factor

Using As=pad in (5), the ultimate moment of resistance is computed as

; = ! (0+) 1 –)1)3!)2! 0

-./ ? = (1 – $\frac{fsp}{fcu} \times \frac{k2}{k1k3}$**)**

&A@ B = 0! ?

Dividing both sides by fcu we get or

$$\frac{Mu}{bd^2} = p \times \frac{fs}{fcu}$$ **× ? – (6)**

A graph plotted between @&AB as shown in fig 2.7 and can be used for design

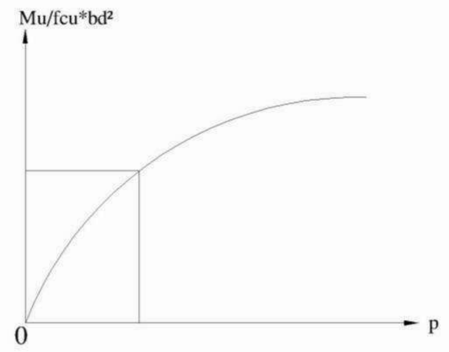

Fig 2.7

2.5 Stress Blocks

Stress blocks adopted by different codes are based on the stress blocks proposed by different investigators. Among them that proposed by Hog nested and Whitney equivalent rectangular block are used by most of the codes. Stress block of IS456-2000 is shown in Fig 2.8. Code recommends ultimate strain ε_{cu}=0.0035 & strain at which the stress reaches design strength ε_0=0.002. Using similar triangle properties on strain diagram

Case 1: Balanced section

Balanced section is considered when the ultimate strain in concrete and in steel are reached simultaneously before collapse.

For equilibrium Cu=Tu

∴ $0.36 f_{ck} x_{umax}\, b = 0.87 f_y Ast_{max}$.

$$\frac{xumax}{d} = \frac{0.87fy}{0.36fck}\,\frac{Astmax}{bd} \text{ but } \frac{Ast\ max}{bd} = pt_{masc}.$$

$$\therefore pt_{max} = \left(\frac{xumax}{d}\right) \times \frac{0.36fck}{0.87fy}$$

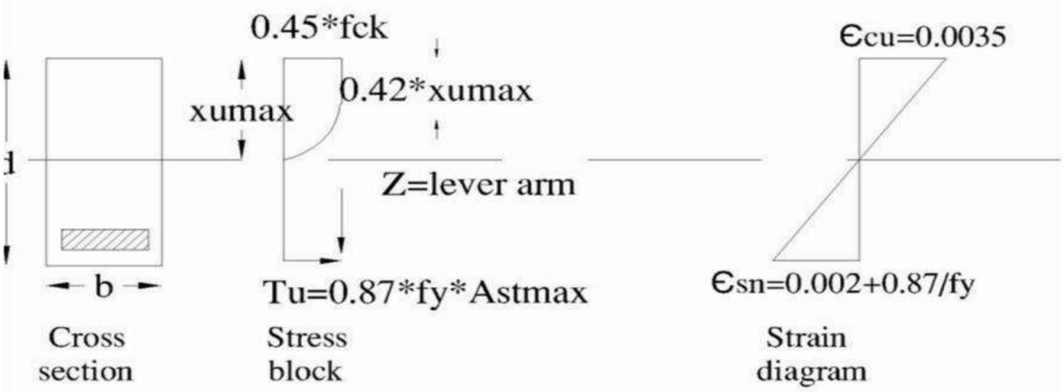

Fig 2.9

Grade of steel this value is given in note of clause 38.1 as (pp70) Values of ':H'A is obtained from equation (12). This value depends on grade of steel. Based on

FyXumax/d

250	0.53
415	0.48
500	0.46(0.456)

Ptmax given in equation (11) is called limiting percentage steel and denoted as pt lim.

To find moment of resistance, the internal moment of Cu &Tu is computed as

Mulim = Cu x Z = $0.36 f_{ck} x_{ulim}$ b (d-0.42 x_{ulim}) from equation (11) ':H'A= 2.42

Mulim = Tu x Z = 0.87fyAst [d-0.42xulim]

Mulim = 0.87fyAst [d-0.42 x 2.42 $-0/: H']

Table 2.1 Limiting Moment resistance & limiting steel

FY	250	415	500
	0.149	0.138	0.133

Where p_{tlim} is in%

Now considering $M_{ulim} = C_u \times Z$.

Mulim = $0.36 f_{ck} x_{ulim}$ b x (d-0.42x_{ulim})

Value of $&A@] I: B is available in table C ~~of SP16~~ & Value o~~f @] I:~~ &AB for different grade of concrete and steel is given in Tables. Value of ptlim for different grade of concrete and steel is given in @] I: &AB Table E of SP –'6'. Term is termed as limiting moment of resistance factor and denoted as Qlim

∴Mulim = $Q_{lim} bd^2$.

Case 2: Under reinforced section

In under reinforced section, the tensile strain in steel attains its limiting value first and at this stage the strain in extreme compressive fiber of concern is less than limiting strain as shown in Fig 2.10

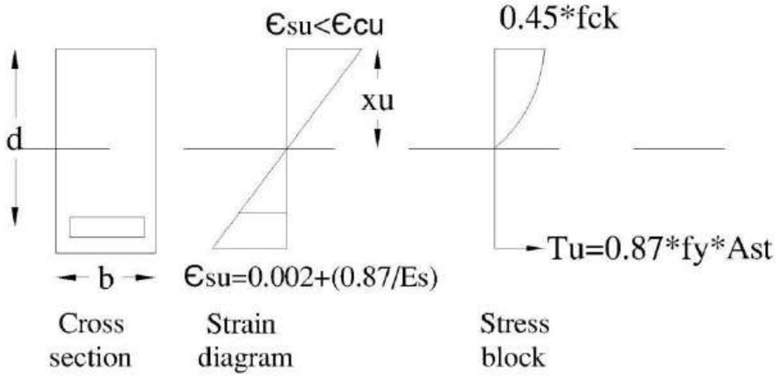

Fig 2.10

The neutral axis depth is obtained from equilibrium condition Cu=Tu

∴ $0.36 f_{ck} x_{ub} = 0.87 fyAst$

xu $= \frac{0.87 fyAst}{0.36 fck\, b} = 2.41 \frac{fy}{fck}$ **(&Q or 'A**

$= 2.41 \frac{fy}{fck} \frac{Ast}{bd} - (16)$

Moment of resistance is calculated considering ultimate tensile strength of steel ∴$M_{uR} = T_u$ x Z or M_{ur} = 0.87fy Ast X (d-0.42xu) = 0.87ftAst d (1-0.42 'A)

= 0.87fyAst d (1-0.42 x 2.41 $ (&AQ)

Considering p_t = 100 (&A expressed as % we get

M_{uR} = 0.87fyAst d(1-1.0122 $\frac{fy}{fck}(\frac{pt}{100})$)

Or $\frac{MuR}{0.87\, fybd^2} = \frac{Ast}{bd}$ (1-1.0122 $\frac{fy}{fck}(\frac{pt}{100})$) , taking 1.0122 ≈ 1

$$\frac{MuR}{0.87\, fybd^2} = \left(\frac{pt}{100}\right) - \frac{fy}{fck}\left(\frac{pt}{100}\right)^2$$

Or $\frac{fy}{fck}\left(\frac{pt}{100}\right)^2 - \frac{pt}{100} + \frac{MuR}{0.87\, fybd^2} = 0 - (17)$

Equation (17) is quadratic equation in terms of (pt/100)

Solving for pt, the value of pt can be obtained as

Pt = 50

Case 3: Over reinforced section in over reinforced section, strain in extreme concrete fiber reaches its ultimate value. Such section fail suddenly hence code does not recommend to design over reinforced section. Depth of neutral axis is computed using equation 4.5-6. Moment of resistance is calculated using concrete strength.

M_{uR} = Cu x Z

= 0.36 fckxub (d-0.42xu) – 19

$\frac{xu}{d} > \frac{xulim}{d}$

Position of neutral axis of 3 cases is compared in Fig. 2.11

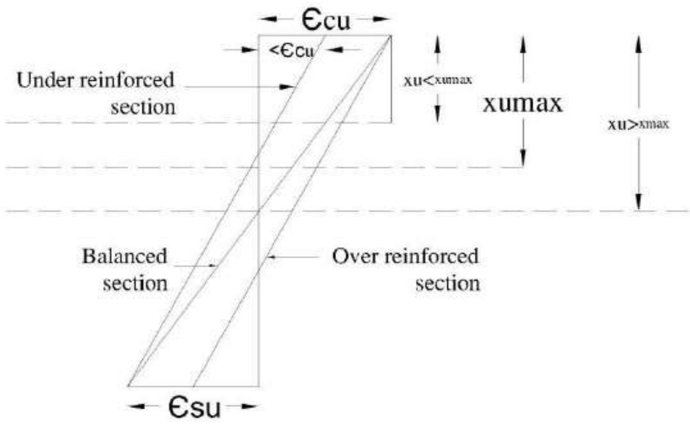

Fig 2.11

